Chapter 10

1

Rotation Of A Rigid Object About A Fixed Axis

1

Lecture 06

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Rolling Objects

2

Rolling Object

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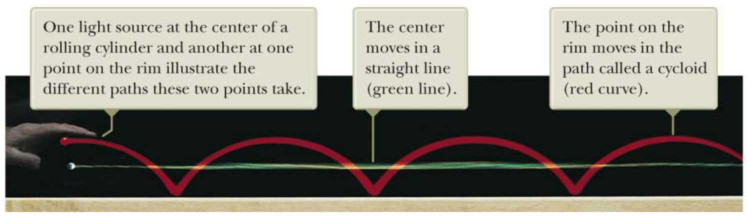
The red curve shows the path moved by a point on the rim of the object.
  This path is called a cycloid.

The green line shows the path of the center of mass of the object.

In pure rolling motion, an object rolls without slipping.

In such a case, there is a simple relationship between its rotational and translational motions.



3

Pure Rolling Motion, Object's Center of Mass

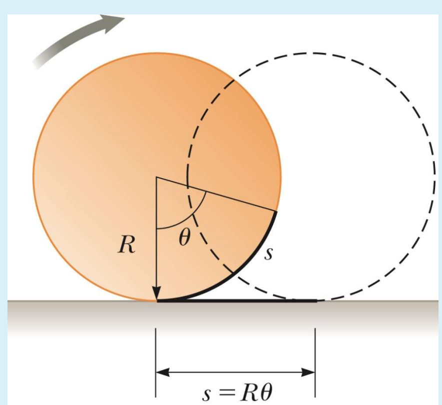
4

The translational speed of the center of mass is

$$v_{com} = \frac{ds}{dt} = R\frac{d\theta}{dt} = R\omega$$

The linear acceleration of the center of mass is

$$a_{com} = \frac{dv_{com}}{dt} = R\frac{d\omega}{dt} = R\alpha$$



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Rolling Motion Cont.

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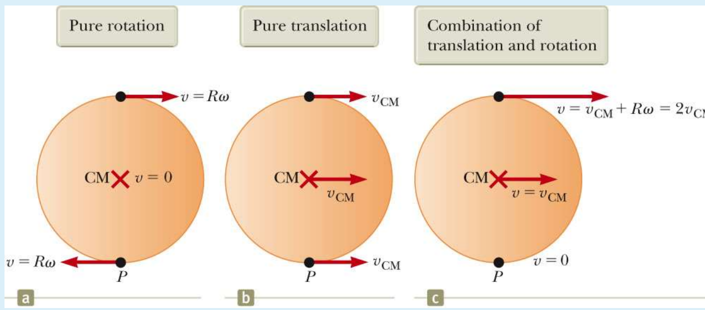
- Rolling motion can be modeled as a combination of pure translational motion and pure rotational motion.
- The contact point between the surface and the cylinder has a translational speed of zero (c).



5

Total Kinetic Energy of a Rolling Object

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The total kinetic energy of a rolling object is the sum of the translational energy of its center of mass and the rotational kinetic energy about its center of mass.

$$K = \frac{1}{2} I_{com}\omega^2 + \frac{1}{2} M v_{com}^2$$

- The $\frac{1}{2} I_{com}\omega^2$ represents the rotational kinetic energy of the cylinder about its center of mass.
- The $\frac{1}{2} M v_{com}^2$ represents the translational kinetic energy of the cylinder about its center of mass.

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Total Kinetic Energy, Example

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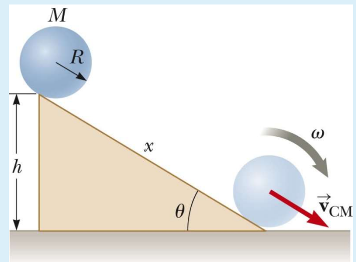
Accelerated rolling motion is possible only if friction is present between the sphere and the incline.

- The friction produces the net torque required for rotation.
- No loss of mechanical energy occurs because the contact point is at rest relative to the surface at any instant.
- In reality, rolling friction causes mechanical energy to transform to internal energy.
  - Rolling friction is due to deformations of the surface and the rolling object.



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Total Kinetic Energy, Example cont.

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Apply Conservation of Mechanical Energy:

- Let $U = U_f = 0$ at the bottom of the plane

$$K_f + U_f = K_i + U_i$$

$$K_f = \frac{1}{2}\frac{I_{com}}{R^2}v_{com}^2 + \frac{1}{2}Mv_{com}^2 = \frac{1}{2}\left(\frac{I_{com}}{R^2} + M\right)v_{com}^2$$

$$U_i = Mgh$$

$$U_f = K_i = 0$$

Solving for v

$$v_{com} = \sqrt{\frac{2gh}{1 + \frac{I_{com}}{MR^2}}}$$

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8

Sphere Rolling Down an Incline

Saturday, 30 January, 2021 16:22

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.

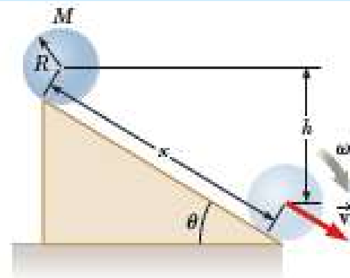
J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.

H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.

H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

For the solid sphere shown:

- calculate the translational speed of the center of mass at the bottom of the incline, and
- the magnitude of the translational acceleration of the center of mass.



$$K = \tfrac{1}{2} I_{CM}\omega^2 + \tfrac{1}{2} M v_{CM}^2$$

$$K = \tfrac{1}{2} I_{CM}\left(\frac{v_{CM}}{R}\right)^2 + \tfrac{1}{2} M v_{CM}^2$$

$$K = \tfrac{1}{2}\left(\frac{I_{CM}}{R^2} + M\right) v_{CM}^2$$

$$\Delta K + \Delta U = 0$$

$$\left[\tfrac{1}{2}\left(\frac{I_{CM}}{R^2} + M\right) v_{CM}^2 - 0\right] + (0 - Mgh) = 0$$

$$v_{CM} = \left[\frac{2gh}{1 + (I_{CM}/MR^2)}\right]^{1/2}$$

$$v_{CM} = \left[\frac{2gh}{1 + (\frac{2}{5}MR^2/MR^2)}\right]^{1/2} = (\tfrac{10}{7}gh)^{1/2}$$

$$h = x \sin\theta.$$

$$v_{CM}^2 = \tfrac{10}{7} g x \sin\theta$$

$$v_{CM}^2 = 2a_{CM}x$$

$$a_{CM} = \tfrac{5}{7} g \sin\theta$$

A disk rolls horizontally

Saturday, 30 January, 2021 16:23

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- [ ] R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
- [ ] J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
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- [x] H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

A disk of mass $M = 1.5\ kg$ and radius $R = 8\ cm$ rolls horizontally without sliding with a center-of-mass speed $v_{com} = 4\ m/s$.

- What is the angular speed of the disk?
- What is the kinetic energy of the rolling disk?

Solution: (a) Using Eq. 8.42, we have:

$$\omega = \frac{v_{\text{CM}}}{R} = \frac{4\,\text{m/s}}{8 \times 10^{-2}\,\text{m}} = 50\,\text{rad/s} \simeq 8\,\text{rev/s}$$

(b) The rolling kinetic energy of the disk is:

$$\begin{aligned} K_{\text{Roll}} &= K_R + K = \tfrac{1}{2} I_{\text{CM}}\,\omega^2 + \tfrac{1}{2} M v_{\text{CM}}^2 \\ &= \tfrac{1}{2}\left(\tfrac{1}{2} M R^2\right)\omega^2 + \tfrac{1}{2} M v_{\text{CM}}^2 \\ &= \tfrac{1}{4}(1.5\,\text{kg}) \times (0.08\,\text{m})^2 (50\,\text{rad/s})^2 + \tfrac{1}{2}(1.5\,\text{kg})(4\,\text{m/s})^2 = 18\,\text{J} \end{aligned}$$

A solid sphere, a disk and a thin hoop

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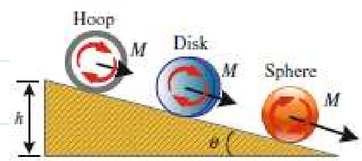
Three objects (a solid sphere, a disk, and a thin hoop) each having a mass M are at rest at the same height h. At the exact same instant, these objects start to roll without sliding down the incline. In what order do they arrive at the bottom?



Solution: For the given list of objects, we set $I_{CM} = \beta M R^2$, where $\beta = 0.4$ for the sphere, $\beta = 0.5$ for the disk, and $\beta = 1$ for the thin hoop. Therefore, using $K_{Roll} = \frac{1}{2} I_{CM}\,\omega^2 = M\,g\,h$ and $v_{CM} = R\omega$, the speed of the center of mass of any one of these objects at the bottom of the incline will be:

$$v_{CM} = \sqrt{\frac{2gh}{\beta + 1}}, \quad \beta = \begin{cases} 0.4 & \text{(for a sphere)} \\ 0.5 & \text{(for a disk)} \\ 1 & \text{(for a hoop)} \end{cases}$$

Note that v_{CM} does not depend on the object's mass M or radius R, but only depends on the shape (through the parameter β) and the height h. Moreover, according to the value of β, the sphere will attain the largest value of v_{CM}, followed by the disk, and finally the hoop will attain lowest value of v_{CM}, see Fig. 8.28.

In all cases, the acceleration of the center of mass is given by:

$$a_{CM} = \frac{g \sin\theta}{(1 + \beta)}$$

This is less than $g \sin\theta$ for the case of a box that slides down a frictionless incline of the same angle.